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DETERMINATION OF THE STRENGTH CHARACTERISTICS OF A PHYSICALLY NONLINEAR INCLUSION IN A LINEARLY ELASTIC MEDIUM

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An isotropic elastic plane with a physically nonlinear inclusion with unknown properties is considered. The general relations between the stress-strain state of the inclusion and the loads applied at infinity are obtained. These relations are used to develop a method of determining the viscoelastoplastic properties of an inclusion that is based on measurement of the displacement vectors of two points that lie on the boundary of the inclusion and are nonsymmetrical with respect to its center. This makes it possible to find numerical values of the constants that enter the constitutive equations of an inclusion.

Eshelby [1] showed that the uniformly distributed stresses applied at infinity cause a uniform stress-strain state in a homogeneous elastic inclusion of given shape. Vakulenko and Sevost'yanov [2] revealed that the stress–strain state of an ellipsoidal inclusion with nonlinear properties which is embedded into an elastic medium is also uniform. This is also true for the two-dimensional case considered in this paper. We propose a method of determining the viscoelastoplastic properties of an elliptic, physically nonlinear inclusion (EPNI) by measuring the displacement vector of two points that lie on the boundary of the inclusion and are nonsymmetrical with respect to its center. The state of the art of engineering techniques makes it possible to perform high-accuracy measurements [3].

1. Stress–Strain State of an Elastic Plane with EPNI. We consider the plane stresses or generalized plane strains of an EPNI-containing isotropic elastic plane under uniformly distributed stresses at infinity (which, generally, depend on the time or loading parameter). We denote the principal values of these stresses by N_1 and N_2, respectively, and the angle made by the first principal axis with the Ox axis by α. The coordinate system Oxy is chosen so that the equation of the boundary L between the elastic medium S and the inclusion S^* has the form $x^2a^{-2} + y^2b^{-2} = 1$, where $a \geqslant b$. Before the stresses are applied at infinity, the regions S and S^* are in the initial undeformed state.

We write Hooke's law for the region S:

$$8\mu\varepsilon_{kl} = (\varkappa - 1)\sigma_{nn}\delta_{kl} + 4\sigma^0_{kl} \quad (k,\, l = 1,\, 2), \quad \sigma^0_{kl} = \sigma_{kl} - \sigma_{nn}\delta_{kl}/2. \tag{1.1}$$

Here σ^0_{kl} and δ_{kl} are the stress-deviator and unit-tensor components, respectively, μ is the shear modulus, $\varkappa = 3 - 4\nu$ refers to plane strain and $\varkappa = (3 - \nu)/(1 + \nu)$ refers to the general plane stress, ν is the Poisson ratio [4]; summation is performed over repeated indices from 1 to 2.

The strains ε_{kl} are assumed to be small and are expressed in terms of the displacements u_k $(k,\, l = 1,\, 2)$ by the Cauchy relations. The load and displacement fields are continuous on the boundary L.

As has already been noted, under the above-indicated conditions, the uniform stress–strain state occurs in the EPNI with nonlinear properties [2]; however, the relations between the stresses and strains in S^* and

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those at infinity are lacking [2]. We assume that the total strains of the EPNI are composed of the elastic ε_{kl}^{e*} and inelastic ε_{kl}^{N*} strains:

$$\varepsilon_{kl}^{*} = \varepsilon_{kl}^{e*} + \varepsilon_{kl}^{N*} \qquad (k, l = 1, 2). \tag{1.2}$$

To construct the desired relations, we map the elastic region S onto the exterior of the unit circle γ in the complex plane ζ [4]:

$$z = \omega(\zeta) = R(\zeta + m\zeta^{-1}), \qquad \zeta = \rho\exp(i\theta),$$
$$2R = a + b, \qquad m = (a - b)/(a + b) \qquad (R > 0, \quad 0 \leqslant m < 1). \tag{1.3}$$

Assuming that the rotation at infinity does not occur and bearing in mind that the principal vector of the forces applied to the boundary L from the side of S^* vanishes (since the stresses σ_{kl}^* at all the points of S^* are equal), we obtain the following expressions for the functions $\varphi(\zeta)$ and $\psi(\zeta)$ that determine the stress–strain state in S [4]:

$$\varphi(\zeta) = \Gamma R\zeta + \varphi_0(\zeta), \quad \psi(\zeta) = \Gamma' R\zeta + \psi_0(\zeta), \quad 4\Gamma = N_1 + N_2, \quad 2\Gamma' = (N_2 - N_1)\exp(-2i\alpha). \tag{1.4}$$

Here $\varphi_0(\zeta)$ and $\psi_0(\zeta)$ are determined from the boundary values of the function $f = 2\partial U/\partial\bar{z}$ [$U = U(z, \bar{z})$ is a stress function] [4, 5]. The continuity condition in the normal direction to the boundary L implies that the equality $\partial U/\partial\bar{z} = \partial U^*/\partial\bar{z}$ must hold on this boundary, i.e.,

$$f = 2\frac{\partial U^*}{\partial\bar{z}} \quad \text{on} \quad L. \tag{1.5}$$

Here $U^* = U^*(z, \bar{z})$ is a stress function for the EPNI.

Since σ_{kl}^* do not depend on x and y (and, hence, on z and $\bar{z}$), with allowance for the relations $\sigma_{11}^* + \sigma_{22}^* = 4\partial^2 U^*/(\partial z\partial\bar{z})$ and $\sigma_{22}^* - \sigma_{11}^* + 2i\sigma_{12}^* = 4\partial^2 U^*/\partial z^2$ [5], after discarding the z and $\bar{z}$ linear terms which have no effect on the stress–strain state, for U^* we obtain

$$2U^* = Az\bar{z} + Bz^2/2 + \bar{B}\bar{z}^2/2, \quad 2A = \sigma_{11}^* + \sigma_{22}^*, \quad 2B = \sigma_{22}^* - \sigma_{11}^* + 2i\sigma_{12}^*. \tag{1.6}$$

From (1.4)–(1.6) and formulas for φ_0 and ψ_0 [4], we find

$$\varphi(\zeta) = R\{\Gamma\zeta + [\bar{B} + m(A - \Gamma) - \bar{\Gamma}']\zeta^{-1}\},$$
$$\psi(\zeta) = R\Big\{\Gamma'\zeta + \frac{[(m^2 + 1)(A - 2\Gamma) + m(B + \bar{B} - \bar{\Gamma}')]\zeta^2 + \bar{B} - m^2 B - \bar{\Gamma}'}{\zeta(\zeta^2 - m)}\Big\}, \quad |\zeta| \geqslant 1. \tag{1.7}$$

Using the relations $2\partial w^*/\partial\bar{z} = \varepsilon_{11}^* - \varepsilon_{22}^* + 2i\varepsilon_{12}^*$ and $\partial w^*/\partial z + \partial\bar{w}^*/\partial\bar{z} = \varepsilon_{11}^* + \varepsilon_{22}^*$, where $w^* = u_1^* + iu_2^*$ [5] (ε_{kl}^* do not depend on z and $\bar{z}$) and assuming that the point $(0, 0) \in S^*$ is fixed, i.e., $w^*(0, 0) = 0$, we express the complex displacement in S^* in the form

$$2w^* = Cz + D\bar{z}, \quad C = \varepsilon_{11}^* + \varepsilon_{22}^* + 2i\varepsilon^*, \quad D = \varepsilon_{11}^* - \varepsilon_{22}^* + 2i\varepsilon_{12}^*, \tag{1.8}$$

where ε^* is an arbitrary constant equal to the rotation value.

Since the displacements at the interface between the regions S and S^* are continuous, i.e., $w = w^*$ on L, the functions $\varphi(\zeta)$ and $\psi(\zeta)$ in (1.7) must satisfy the known boundary condition for displacements; the latter can be combined with a similar condition for the stress problem [4, 5] and relations (1.8) to give

$$(æ + 1)\overline{\varphi(\sigma)} = (A + \mu\bar{C})\overline{\omega(\sigma)} + (B + \mu\bar{D})\omega(\sigma), \tag{1.9}$$

where $\sigma = \exp(i\theta)$ is an arbitrary point on γ.

Substituting (1.3) and (1.7) into (1.9) and equating the coefficients of σ and σ^{-1}, we obtain

$$\mu(m\bar{C} + \bar{D}) = æ(mA + B) - (æ + 1)(m\Gamma + \Gamma'),$$
$$\mu(\bar{C} + m\bar{D}) = -(A + mB) + (æ + 1)\Gamma. \tag{1.10}$$

Using the expressions for A, B, C, and D in (1.6) and (1.8) and the equalities $\sigma_{11}^{\infty} + \sigma_{22}^{\infty} = 4\Gamma$ and $\sigma_{22}^{\infty} - \sigma_{11}^{\infty} + 2i\sigma_{12}^{\infty} = 2\Gamma'$ [4] and performing simple manipulations, we obtain from (1.10) the desired relations between the stresses and strains in S^* and those at infinity:

$$F_i = \alpha_{ij} y_j + \beta_{ij} x_j \qquad (i = 1, 2, 3),$$

$$F_1 = \varepsilon_{11}^*, \quad F_2 = \varepsilon_{22}^*, \quad F_3 = 2\varepsilon_{12}^*, \quad y_1 = \sigma_{11}^*, \quad y_2 = \sigma_{22}^*, \quad y_3 = \sigma_{12}^*,$$

$$x_1 = \sigma_{11}^{\infty}, \quad x_2 = \sigma_{22}^{\infty}, \quad x_3 = \sigma_{12}^{\infty}, \quad \alpha_{11} = -\frac{(æ + 1)(1 - m)}{4\mu(1 + m)},$$

$$\alpha_{12} = \alpha_{21} = \frac{æ - 1}{4\mu}, \quad \alpha_{22} = -\frac{(æ + 1)(1 + m)}{4\mu(1 - m)}, \quad \alpha_{33} = -\frac{æ + m^2}{\mu(1 - m^2)}, \tag{1.11}$$

$$\beta_{11} = \frac{(æ + 1)(3 - m)}{8\mu(1 + m)}, \quad \beta_{12} = \beta_{21} = -\frac{æ + 1}{8\mu}, \quad \beta_{22} = \frac{(æ + 1)(3 + m)}{8\mu(1 - m)},$$

$$\beta_{33} = \frac{æ + 1}{\mu(1 - m^2)} \qquad (0 \leqslant m < 1),$$

where the other coefficients α_{ij} and β_{ij} are equal to zero; summation in (1.11) is performed over j from 1 to 3.

The constitutive equations for S^* of the form (1.2) and (1.11) form a closed system that allows us to determine the loading history $\sigma_{kl}^* = \sigma_{kl}^*(t)$ $(k, l = 1, 2)$ in the EPNI from the known loading history $\sigma_{kl}^{\infty} = \sigma_{kl}^{\infty}(t)$ at infinity, where t is the time or loading parameter.

One can show that the system is uniquely solvable for $\sigma_{kl}^* = \sigma_{kl}^*(t)$ under the usual assumptions of stable inelastic deformation [6]. Thus, if $\varepsilon_{kl}^{N*} = \varepsilon_{kl}^{N*}(\sigma_{mn}^*)$ are differentiable functions (which is, for example, the case where the behavior of the inclusion material is described by the deformation theory of plasticity), the stability condition has the form $\Delta\varepsilon_{kl}^{N*}\Delta\sigma_{kl}^* \geqslant 0$. If ε_{kl}^{N*} are the irreversible strains (plastic, viscous, and creep strains) satisfying relations of the flow theory, a similar inequality has the form $\int\limits_0^t \Delta\dot{\varepsilon}_{kl}^{N*}\Delta\sigma_{kl}^*\,dt \geqslant 0$ at any moment $t > 0$ for the corresponding initial conditions at $t = 0$ [6].

One can show that the above solution for an EPNI-containing plane is unique, i.e., for given σ_{kl}^{∞} and the above-indicated restrictions imposed on the relation between ε_{kl}^{N*} and σ_{kl}^*, the stress–strain state occurring in S^* is uniform.

If the uniform strains ε_{kl}^{N*} are specified in S^* (whose nature is of no significance) and $\sigma_{kl}^{\infty} = 0$, the stresses σ_{kl}^* are determined from relations (1.2) and (1.11) uniquely. This case was studied by Eshelby [1].

2. Determination of the Elastoviscoplastic Characteristics of an Inclusion. We assume that the elastic constants μ and $æ$ of the medium S and the geometrical parameters R and m of the inclusion S^* are known. It is required to determine the mechanical characteristics of the inclusion. With the use of (1.11), one can solve this problem or, to be more specific, the problem of determining the numerical values of the constants in Eqs. (1.2), in which the relations for ε_{kl}^{e*} and ε_{kl}^{N*} are to be detailed. We assume that the elastic strains obey Hooke's law, i.e., ε_{kl}^{e*} are expressed in terms of σ_{kl}^* by linear relations which involve six unknown constants in the two-dimensional case. The inelastic strains ε_{kl}^{N*} are composed of the plastic strains ε_{kl}^{p*} and the time-dependent viscous strains ε_{kl}^{c*}: $\varepsilon_{kl}^{N*} = \varepsilon_{kl}^{p*} + \varepsilon_{kl}^{c*}$ $(k, l = 1, 2)$. With allowance for this relation, we write (1.2) in the form

$$F_i = \alpha_{ij}^e y_j + f_i^p + f_i^c \qquad (i = 1, 2, 3), \tag{2.1}$$

$$f_1^p = \varepsilon_{11}^{p*}, \quad f_2^p = \varepsilon_{22}^{p*}, \quad f_3^p = 2\varepsilon_{12}^{p*}, \quad f_1^c = \varepsilon_{11}^{c*}, \quad f_2^c = \varepsilon_{22}^{c*}, \quad f_3^c = 2\varepsilon_{12}^{c*},$$

where $\|\alpha_{ij}^e\|$ $(i, j = 1, 2, 3)$ is the symmetrical matrix composed of the elastic-compliance components of the inclusion, i.e., the elastic strains of the inclusion f_i^e are given by

$$f_i^e = \frac{1}{2}\frac{\partial s_e^2}{\partial y_i}, \qquad s_e^2 = \alpha_{ij}^e y_i y_j. \tag{2.2}$$

To determine the elastoplastic properties, we assume that the external loads, i.e., the stresses x_i, vary in direct proportion with one parameter. This allows us to use (at least, in the first approximation) the relations of the deformation theory of plasticity:

$$f_i^p = \begin{cases} 0, & s_p < \sigma_Y, \\ \lambda \dfrac{\partial s_p}{\partial y_i}, & s_p \geqslant \sigma_Y. \end{cases} \tag{2.3}$$

Here $s_p = s_p(y_i)$ is a first-power homogeneous function, σ_Y is the yield point upon uniaxial extension along the Ox axis, $\lambda > 0$ is the undetermined multiplier for an ideal plastic material [in this case, the second inequality in (2.3) is replaced by the equality $s_p = \sigma_Y$], and $\lambda = \lambda(s_p)$ is a monotonically increasing function for a hardening material, which is assumed to be a power function:

$$\lambda = B_0 s_p^q. \tag{2.4}$$

We write s_p by analogy with relation (2.2) for s_e:

$$s_p^2 = \alpha_{ij}^p y_i y_j, \qquad \alpha_{ij}^p = \alpha_{ji}^p, \qquad \alpha_{11}^p = 1. \tag{2.5}$$

It is noteworthy that the function s_p in (2.5) is a generalization of the relations $s_p = s_p(y_i)$ for an isotropic medium for $\alpha_{11}^p = \alpha_{22}^p = 1$, $\alpha_{12}^p = 1 - \beta/2$, and $\alpha_{33}^p = \beta$, and the other α_{ij}^p are zero (the Mises criterion for $\beta = 3$ and the Tresca criterion for $\beta = 4$.)

Finally, in determining the viscous strains f_i^c, we assume that the aftereffects do not occur in the EPNI (which can be verified experimentally) and use the common relations of the steady-creep theory [6]

$$\dot{f}_i^c = s_c^n \frac{\partial s_c}{\partial y_i}, \qquad s_c^2 = \alpha_{ij}^c y_i y_j, \qquad \alpha_{ij}^c = \alpha_{ji}^c. \tag{2.6}$$

Instead of the power function in (2.6), one can use other functions, for example, exponential, hyperbolic sine, fraction-linear, etc. [6].

We assume that when the external stresses x_i are applied, the displacements u_k^* ($k = 1, 2$) of two points L located on the boundary nonsymmetrically relative to the EPNI center [which is assumed to be fixed: $w^*(0,0) = 0$] can be measured. As follows from (1.8) and (1.3), the strains F_i and the rotation of ε^* are determined uniquely in the inclusion. Indeed, substituting the values of $z_k = R(\sigma_k + m\sigma_k^{-1})$ and $\bar{z}_k = R(\sigma_k^{-1} + m\sigma_k)$ ($k = 1, 2$) into (1.8), we obtain a system of two linear equations for C and D whose determinant $R^2(1 - m^2)(\sigma_1\sigma_2^{-1} - \sigma_2\sigma_1^{-1})$ does not vanish for $z_1 \neq z_2$ and $z_1 \neq -z_2$.

Given the strains in the inclusion F_i and the external stresses x_i, the stresses in the inclusion y_i ($i = 1, 2, 3$) are determined uniquely, since, according to (1.11), the matrix $\|\alpha_{ij}\|$ is negative definite and $\det \|\alpha_{ij}\| < 0$.

Bearing the foregoing in mind, we propose the following algorithm of calculating the constants in the constitutive equations.

Determination of the Elastic Constants α_{ij}^e. It is necessary to measure the displacements of the above-indicated points, from which the strains F_i are determined for combinations of the external loads $x_i^{(k)}$ such that the corresponding stresses $y_i^{(k)}$ in (1.11) are linearly independent, i.e., $\Delta \equiv \det \|y_i^{(k)}\| \neq 0$, and do not cause the plastic strains $f_i^{p(k)}$ ($i, k = 1, 2, 3$) in the inclusion. The latter can be verified by unloading, i.e., removing the loads $x_i^{(k)}$ and measuring the residual displacements $\tilde{w}^{*(k)}$ of the above-indicated points of the contour L (if $f_i^{p(k)} = 0$, the condition $\tilde{w}^{*(k)} = 0$ must hold and vice versa). It is assumed that the loads are applied at infinity instantaneously (the viscous strains $f_i^{c(k)}$ do not occur in the inclusion).

With allowance for these remarks, it follows from (2.1) that

$$\alpha_{ij}^e y_j^{(k)} = F_i^{(k)} \qquad (i,\ k = 1, 2, 3). \tag{2.7}$$

Relations (2.7) constitute a system of nine linear equations for six desired α^e_{ij}. If we confine our analysis to two experiments ($k = 1, 2$), the matrix of the system is degenerate despite the fact that the number of unknowns coincides with the number of equations in (2.7). Indeed, for the desired vector $\{\alpha^e_{11}, \alpha^e_{12}, \alpha^e_{13}, \alpha^e_{22}, \alpha^e_{23}, \alpha^e_{33}\}$, the structure of the matrix $\|a_{ij}\|$ ($i, j = 1, 2, \ldots, 6$) is as follows: $a_{11} = a_{32} = a_{53} = y^{(1)}_1$, $a_{12} = a_{34} = a_{55} = y^{(1)}_2$, $a_{13} = a_{35} = a_{56} = y^{(1)}_3$, $a_{21} = a_{42} = a_{63} = y^{(2)}_1$, $a_{22} = a_{44} = a_{65} = y^{(2)}_2$, and $a_{23} = a_{45} = a_{66} = y^{(2)}_3$, with other a_{ij} being equal to zero. Direct calculations show that $\det\|a_{ij}\| = 0$. Therefore, setting $i = 1$, we find α^e_{1j} from three equations corresponding to $k = 1$, 2, and 3. Similarly, setting $i = 2$ and then $i = 3$, we obtain systems for α^e_{2j} and α^e_{3j} ($j = 1, 2, 3$) for $k = 1$, 2, and 3 whose determinants do not vanish for the three cases ($i = 1, 2, 3$). Consequently, all the constants α^e_{ij} are determined uniquely.

It should be noted that the coefficients α^e_{ij} and α^e_{ji} ($i \neq j$) are determined by the above method as independent coefficients. Since $\alpha^e_{ij} = \alpha^e_{ji}$, a comparison between these quantities can serve as a criterion to check the measurement accuracy and the validity of the adopted hypotheses, in particular, the hypothesis that the stresses and elastic strains in the inclusion are related linearly.

We consider the case of an isotropic inclusion. Let the equality $2y_3/(y_1 - y_2) = F_3/(F_1 - F_2)$ be sufficiently accurate for the data obtained in the above-described experiments, which shows indirectly that the stress and strain tensors are coaxial in the two-dimensional case: if this equality holds for any combinations of y_i ($i = 1$, 2, 3), the elastic constants are related, by virtue of (2.1) (for $f^p_i = f^c_i = 0$), by the formulas $\alpha^e_{11} = \alpha^e_{22}$, $\alpha^e_{33} = 2(\alpha^e_{11} - \alpha^e_{12})$, and $\alpha^e_{13} = \alpha^e_{23} = 0$, which corresponds to Hooke's law for an isotropic medium of the form (1.1) that contains only two constants $\mu^* = (1/2)(\alpha^e_{11} - \alpha^e_{12})^{-1}$ and $æ^* = (3\alpha^e_{11} + \alpha^e_{12})(\alpha^e_{11} - \alpha^e_{12})^{-1}$. Therefore, in the first approximation, for elastic strains, the inclusion can be assumed to be isotropic, and α^e_{11} and α^e_{12} can be determined from the data obtained in one of the above-mentioned experiments, for example, from the relations $\alpha^e_{11}y_1 + \alpha^e_{12}y_2 = F_1$ and $\alpha^e_{12}y_1 + \alpha^e_{11}y_2 = F_2$ provided $|y_1| \neq |y_2|$.

Determination of Plastic Characteristics. Assuming that the quantities α^e_{ij} are known, for elastic deformation of the EPNI, we obtain the following equalities, which follow from (1.11) and (2.1):

$$A_{ij}y_j = \beta_{ij}x_j, \qquad A_{ij} = \alpha^e_{ij} - \alpha_{ij} \qquad (i, j = 1, 2, 3). \tag{2.8}$$

It is seen from (1.11) that the matrices $\|A_{ij}\|$ and $\|\beta_{ij}\|$ are positive definite and, hence, can be inverted. This implies that the relations between y_i and x_j are unique ($i, j = 1$, 2, 3). Consequently, the required loading program can be realized in the inclusion (in the elastic region). For example, if it is required to ensure a uniaxial stress state characterized by only one nonzero component y_k, it follows from (2.8) that the external loads must be of the form $x_i = B_{ik}y_k$ ($i = 1$, 2, 3; no summation over k), where $B_{ik} = \beta^{-1}_{ij}A_{jk}$ (β^{-1}_{ij} are the components of the matrix inverse to $\|\beta_{ij}\|$).

Let us assume that upon carrying out three independent loading programs, the moment of onset of the plastic strains f^p_i (more precisely, their rates $\dot{f}^p_i$) was recorded for each program. The quantities f^p_i and y_i are determined from (1.11) and (2.1) for known values of F_i and x_i. From relations (2.3), we obtain

$$f^p_i = A_p s_p^{-2} \alpha^p_{ij} y_j \qquad (i = 1, 2, 3), \qquad A_p = f^p_i y_i. \tag{2.9}$$

It should be noted that at this moment, the equality $s_p = \sigma_Y$ holds for each of the three experiments. With this in mind, we obtain from (2.9) a system of the form (2.7) for determining six unknowns (α^p_{ij} and σ_Y), where α^e_{ij} should be replaced by $\alpha^p_{ij}\sigma_Y^{-2}$ and $F^{(k)}_i$ by $f^{p(k)}_i/A^{(k)}_p$. The resulting system is solved by the technique outlined above.

If the EPNI material hardens in the plastic deformation, i.e., $\dot{s}_p > 0$ for further active loading at infinity, the values of A_p and s_p obtained in two experiments for $s_p > \sigma_Y$ and $s^{(1)}_p \neq s^{(2)}_p$ suffice to determine the constants B_0 and q in (2.4), since $A_p = B_0 s_p^{q+1}$ according to (2.3) and (2.4). Given $F^{(k)}_i$, $x^{(k)}_i$, and α^p_{ij}, the values of $A^{(k)}_p$ and $s^{(k)}_p$ ($k = 1$, 2) are readily determined from (1.11), (2.1), (2.5), and (2.9). In this case, it is assumed that the viscous strains do not occur in the elastoplastic deformation of the EPNI, i.e., $f^c_i = 0$.

Determination of Viscous Characteristics. The technique of determining the constants α^c_{ij} and n in (2.6) is similar to the above-described technique of determining the elastoplastic characteristics. It suffices to perform three independent experiments and measure $F_i^{(k)}(t)$ and $\dot{F}_i^{(k)}(t_0)$ for the external loads $x_i^{(k)} = x_i^{(k)}(t)$ such that the corresponding stresses $y_i^{(k)}(t)$ in (1.11) are subject to the conditions $\det\|y_i^{(k)}(t_0)\| \neq 0$ and $s_p(y_i^{(k)}(t)) < \sigma_Y$, i.e., $f_i^{p(k)}(t) = 0$ ($i, k = 1, 2, 3$; $0 < t \leqslant t_0$). Here t_0 is the moment chosen for observation (at $t < 0$, the elastic medium S and the inclusion S^* are in their initial states). The quantities $\dot{f}_i^{c(k)}(t_0)$ are determined from (2.1).

It follows from (2.6) that the specific dissipation power $W = \dot{f}_i^c y_i$ satisfies the equality $W = s_c^{n+1}$, i.e., $s_c = W^{1/(n+1)}$. As a result, relation (2.6) can be written in the form

$$\dot{f}_i^c = s_c^{n-1}\alpha_{ij}^c y_j = W^{(n-1)/(n+1)}\alpha_{ij}^c y_j.$$

At the moment $t = t_0$, the quantities $\dot{f}_i^c$ and W are known; therefore, assuming the creep exponent n to be known, we obtain the following system to determine α_{ij}^c, which is similar to system (2.7):

$$\alpha_{ij}^c y_j^{(k)} = W_k^{(1-n)/(n+1)}\dot{f}_i^{c(k)} \qquad (i, k = 1, 2, 3). \tag{2.10}$$

Since $\alpha_{ij}^c = \alpha_{ji}^c$, system (2.10) becomes

$$\alpha_{ij}^c y_i^{(1)} y_j^{(2)} = W_1^{(1-n)/(n+1)}\dot{f}_i^{c(1)} y_i^{(2)} = W_2^{(1-n)/(n+1)}\dot{f}_i^{c(2)} y_i^{(1)}.$$

Provided $W_1 \neq W_2$, n is determined uniquely from this equation. Then, system (2.10) is solved for α_{ij}^c by the same technique as (2.7).

In concluding, the results obtained can be generalized to the case of a linear viscoelastic medium with an EPNI characterized by more complex constitutive equations compared to (2.1)–(2.6), which allow for the aftereffects, hardening and unhardening, accumulation of defects in creep, etc.

REFERENCES

1. J. D. Eshelby, *Proc. Roy. Soc. London*, **A241**, No. 1226, 337 (1957).
2. A. A. Vakulenko and I. B. Sevost'yanov, "An inclusion with nonlinear properties in an elastic medium," in: *Mechanics of Building Structures and Materials* [in Russian], Leningr. Inzh.-Str. Ist., Leningrad (1991), pp. 8–16.
3. W. Ranson, M. Sutton, and W. Peters, "Holographic and laser speckle interferometry," in: *Experimental Mechanics* [Russian translation], Vol. 1, Mir, Moscow (1990), pp. 448–491.
4. N. I. Muskhelishvili, *Some Basic Problems of the Mathematical Theory of Elasticity*, Noordhoff, Groningen, Holland (1953).
5. G. N. Savin, *Stress Distribution near Holes* [in Russian], Naukova Dumka, Kiev (1968).
6. I. Yu. Tsvelodub, *Stability Postulate and Its Application to the Creep of Metals* [in Russian], Inst. of Hydrodynamics, Sib. Div., Russian Acad. of Sci., Novosibirsk (1991).